https://en.wikipedia.org/wiki/Manifold

# Manifold introduction

## ||Definition||

### A manifold is a topological space that locally resembles Euclidean space near each point

## ||Examples||

### One-dimensional

##### Lines and circles

##### Not figure eights

Because they crossing point

At the point is not locally homeomorphic to Euclidean 1-space

### Two-dimensional (surfaces)

##### Embedded in 3D real space

Plane, sphere, torus

##### Cannot be embedded in 3D real space

Klein bottle

Real projective plane

## ||Related to||

### Geometry

### Modern mathematical physics

### It allows more complicated structures to be described and understood in terms of the relatively well-understood properties of Euclidean space

## ||Arise as||

### Solution sets of systems of equations

### Graphs of functions

## ||Additional features||

### Differentiable manifolds

##### Differentiable structure allows calculus to be done on manifolds

### Riemannian metric

##### Allows distances and angles to be measured

### Simplistic manifolds (辛流形)

##### Serve as the phase spaces in the Hamiltonian formalism of classical mechanics

### Four-dimensional Lorentzian manifolds

##### Model spacetime in general relativity

# Motivational examples

### Circle

##### For unit circle

##### Four charts

##### Transition map

##### *….*

### Enrich circle

##### 

### Sphere

##### May be covered by an atlas of six charts

##### …

### Other curves

##### Topological operations always preserve the number of pieces

# Euler characteristic

### Betti number

# Gauss–Bonnet theorem

# Synthesis

### Complex manifold

### Bernhard Riemann

# Mathematical definition

### Informally, a manifold is a space that is "modeled on" Euclidean space

### Topological manifold

##### A topological manifold is a second countable Hausdorff space that is locally homeomorphic to Euclidean space

##### Locally homeomorphic to Euclidean space means that every point has a neighborhood homeomorphic to an open Euclidean n-ball

Manifolds are taken to have a fixed dimension (the space must be locally homeomorphic to a fixed n-ball), and such a space is called an n-manifold

Pure manifold

Not pure

### Broad definition

##### The broadest common definition of manifold is a topological space locally homeomorphic to a topological vector space over the reals

Not topological

Hilbert manifolds

Banach manifolds

Fréchet manifolds

# Charts, atlases, and transition maps

## ||Charts||

### An invertible map between a subset of the manifold and a simple space such that both the map and its inverse preserve the desired structure

##### For a topological manifold

The simple space is some Euclidean space

This structure is preserved by homeomorphisms

Invertible maps that are continuous in both directions

##### In the case of a differentiable manifold

A set of charts called an atlas allows us to do calculus on manifolds

Polar coordinates, for example, form a chart for the plane minus the positive x-axis and the origin

## ||Atlases||

### A specific collection of charts which covers a manifold is called an atlas

##### Not unique

### Two atlases are said to be -equivalent if their union is also a atlas

### The atlas containing all possible charts consistent with a given atlas is called the maximal atlas

##### Unique

## ||Transition maps||

### Given two overlapping charts, a transition function can be defined which goes from an open ball in to the manifold and then back to another (or perhaps the same) open ball in

## ||Additional structure||

### If all the transition maps are compatible with this structure, the structure transfers to the manifold

##### Differentiable manifold

If the transition functions of an atlas for a topological manifold preserve the natural differential structure of

##### Complex manifold

Introduced in an analogous way by requiring that the transition functions of an atlas are holomorphic functions

##### Symplectic manifolds

The transition functions must be symplectomorphisms

# Manifold with boundary

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